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Technical Note

Thermal convection in a vertical porous slot with spatially periodic boundary temperatures: low *Ra* flow

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Abstract

This note considers steady two-dimensional thermal convection in a porous layer between two infinite-vertical walls kept at spatially periodic temperatures. The spatial non-uniformities of the temperature distributions are assumed to be small, and an analytical solution is found. The effects of the wave number and the phase difference of the wall temperatures on the flow are investigated. Maximum heat transfer occurs at the wave number of 1.606 and a phase difference of $\pi/2$.

1. Introduction

Thermal convection in porous media has received much attention because of the theoretical interest and its wide applications ranging from engineering to geophysics such as heat insulation by fibrous materials, spreading of pollutants, solar energy collectors, and geothermal energy analysis. Comprehensive reviews on the convection in porous media were presented in the recent monographs by Nield and Bejan [1], Vafai [2], Ingham and Pop [3], and Pop and Ingham [4]. Much works have been made for the convection with constant temperature boundary conditions, but relatively few studied the thermal convection in porous media with boundary surfaces having non-uniform temperatures [5–8].

This note considers a 2-D fluid saturated porous slot between two infinite-vertical walls kept at spatially periodic temperatures (Fig. 1). The left and right walls have temperature distributions of $T_L = T_0 + \Delta T \sin\left(k\frac{X}{L}\right)$ and $T_R = T_0 + \Delta T \sin\left(k\frac{X}{L} - \beta\right)$ with $k > 0$, respectively. The present configuration is motivated by the building insulation system using porous medium [9,10]. The object of this study is to investigate the effect of spatial non-uniformities on the flow and heat transfer characteristics. The wave number (k) and the phase difference (β) represent the frequency and the stagger of arrangement of the non-uniform temperatures on the walls, respectively. Although the present configuration is very simple, we can see the qualitative features of the thermal convection depending on the wave number and the phase difference of the spatial temperature non-uniformities on the walls.

2. An analytical solution for small *Ra*

The dimensionless Darcy–Boussinesq equations [5] governing 2-D steady-state natural convection in an isotropic porous medium are given by

$$\nabla^2 \Psi = -Ra \frac{\partial \theta}{\partial y} \tag{1}$$

$$\nabla^2 \theta = -\frac{\partial \theta}{\partial x}\frac{\partial \Psi}{\partial y} + \frac{\partial \theta}{\partial y}\frac{\partial \Psi}{\partial x} \tag{2}$$

where the streamfunction Ψ is defined as

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \tag{3}$$

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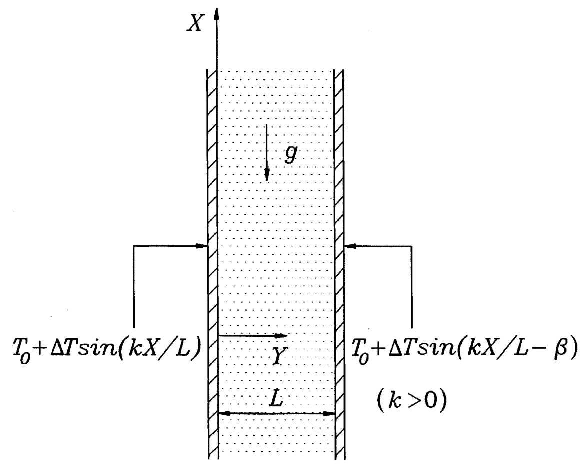

Fig. 1. A vertical porous layer bounded by two impermeable walls with spatially periodic temperature distributions.

and θ is the dimensionless temperature: $\theta = (T - T_0)/\Delta T$. The Darcy-modified Rayleigh number (Ra) is defined as $Ra = Kg\alpha L\Delta T/\kappa\nu$, where K is the permeability of the porous matrix, α and ν are the thermal expansion coefficient and the kinematic viscosity of the fluid, respectively, κ is the effective thermal diffusivity of the saturated porous medium, and g is the gravitational acceleration. The characteristics length is taken as the thickness of the porous layer (L).

The dimensionless boundary conditions are

$$\Psi = 0 \quad \text{at } y = 0, 1 \tag{4}$$

$$\theta = \sin(kx) \quad \text{at } y = 0 \tag{5}$$

$$\theta = \sin(kx - \beta) \quad \text{at } y = 1 \tag{6}$$

Since the wall temperatures are periodic, we consider the averaged heat transfer on the walls over one wave length $2\pi/k$ ($k \neq 0$). The mean Nusselt number ($\overline{Nu}$) is thus defined as

$$\overline{Nu} = -\frac{k}{2\pi}\int_0^{2\pi/k} \frac{\partial\theta}{\partial y}\,\mathrm{d}x \quad \text{at } y = 0, 1 \tag{7}$$

The solutions of Eqs. (1)–(6) for small Ra are found by expanding $\Psi(x,y)$ and $\theta(x,y)$ as

$$\Psi(x,y) = \Psi_0(x,y) + Ra\Psi_1(x,y) + \cdots$$
$$\theta(x,y) = \theta_0(x,y) + Ra\theta_1(x,y) + \cdots \tag{8}$$

Introducing (8) to Eqs. (1) and (2) and solving the resulting equations for like power in Ra subject to the corresponding boundary conditions, we obtain

$$\Psi_0(x,y) = 0, \quad \theta_0(x,y) = f(1-y)\sin(kx) + f(y)\sin(kx-\beta) \tag{9}$$

$$\Psi_1(x,y) = -g(1-y)\sin(kx) + g(y)\sin(kx-\beta) \tag{10}$$

where

$$f(y) = \frac{\sinh(ky)}{\sinh(k)} \quad \text{and}$$
$$g(y) = \frac{(1-y)}{2\sinh(k)}\sinh(ky) \tag{11}$$

The zeroth-order solution (Ψ_0, θ_0) corresponds to the pure conduction state without fluid flow, and the mean Nusselt number ($\overline{Nu}$) of it is zero. But the fluid flow in the porous layer can give rise to non-zero averaged heat transfer at the walls, and $\overline{Nu}$ of the first order temperature distribution is obtained as

$$\overline{Nu} = Ra[Nu(k)\sin(\beta)] \tag{12}$$

where

$$Nu(k) = \frac{k\cosh(k) - \sinh(k)}{8\sinh^2(k)} \tag{13}$$

3. Results and discussion

The variation of flow and temperature fields with respect to β ($0 \leqslant \beta \leqslant \pi$) is shown in Fig. 2 with $k = 3.1$. The present system posses a point symmetry of $\Psi(\beta/k - x, 1 - y) = \Psi(x,y)$ and $\theta(\beta/k - x, 1 - y) = -\theta(x,y)$. And the characteristics of the flow at $\beta^* = 2\pi - \beta(\pi \leqslant \beta^* \leqslant 2\pi)$ is identical to that of β ($0 \leqslant \beta \leqslant \pi$). The flow of $\beta = 0$ (Fig. 2(a)) consists of four closed rolls rotating clockwise and counter-clockwise directions over one wave length ($0 \leqslant x \leqslant 2\pi/k$), where the fluid near the points of maximum and minimum wall temperatures moves upward and downward, respectively. As β increases, the two co-rotating eddies are merged in a new cell, and at $\beta = \pi$ (Fig. 2(f)) two counter-rotating eddies are formed over one period of the wall temperatures. Fig. 2(a)–(f) shows smooth transition of flow fields from four-eddy to two-eddy pattern, as β increases from 0 to π.

The flows of small k ($\leqslant 3$) are similar to those in Fig. 2. As k becomes large, the thermal interaction between two walls is gradually decreased. And at $k \geqslant 5$, two co-rotating eddies appear in a cell (Fig. 3) for all β except $\beta = 0$ where two counter-rotating eddies appear in a cell. Fig. 3 presents the flows of $k = 6.3$ and 12.6 with $\beta = \pi/2$ and π. The flows of a large $k(= 12.6)$ in Fig. 3 show that the fluid in the central part is almost stagnant, and the formation of the eddies near the one wall which are nearly unaffected by the eddies near the other wall. The transition of flows with respect to k is also smooth.

The mean Nusselt number ($\overline{Nu}$) in Eq. (12) shows the dependency of heat transfer rate on the wave number (k) and phase difference (β). As can be readily seen, $\overline{Nu} \propto \sin(\beta)$. On the other hand, Riley [10] considered a vertical slot filled with a fluid-saturated porous medium contained between two small amplitude wavy surfaces: the lateral surfaces (y_1, y_2) were defined by $y_1 = -1 - \varepsilon\cos(kx + \gamma)$ and $y_2 = 1 + \varepsilon\cos(kx - \gamma)$ with

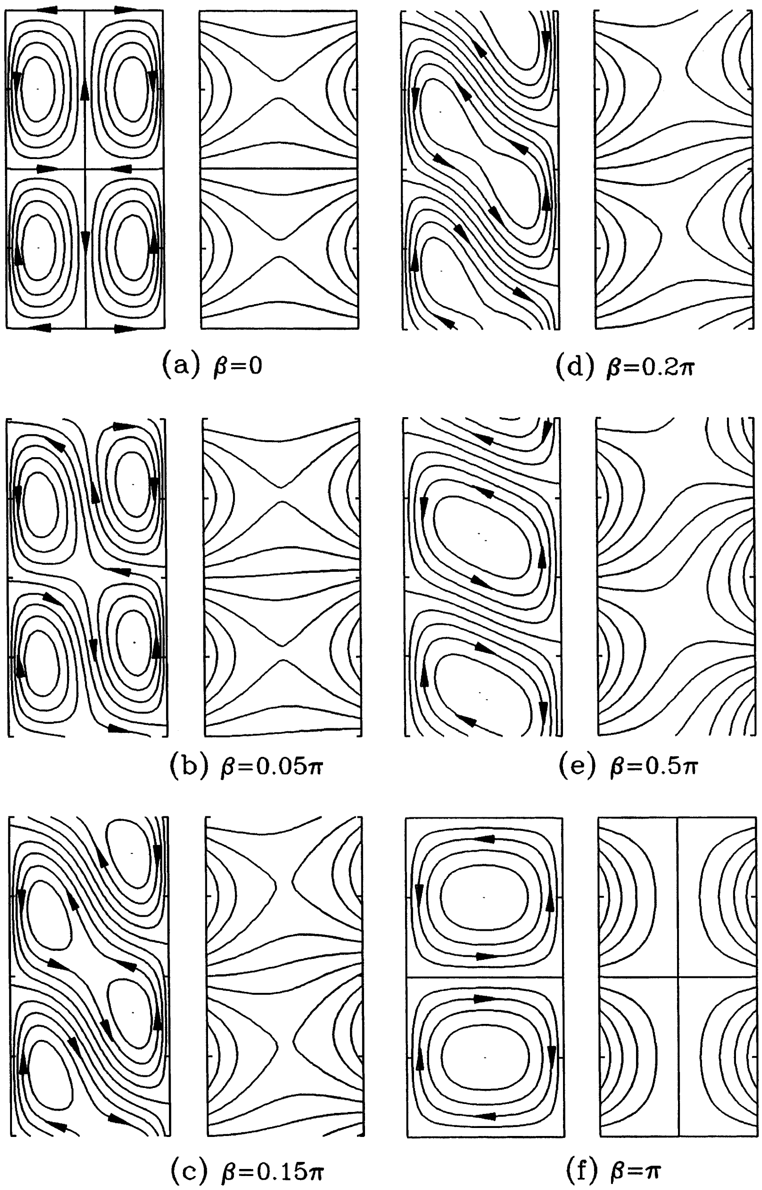

Fig. 2. Variation of flow and temperature fields with respect to β for $k = 3.1$: (a) $\beta = 0$; (b) $\beta = 0.05\pi$; (c) $\beta = 0.15\pi$; (d) $\beta = 0.2\pi$; (e) $\beta = 0.5\pi$; (f) $\beta = \pi$. The range of x is $0 \leqslant x \leqslant 2\pi/k$.

$\varepsilon \ll 1$. He showed that maximum and minimum heat transfer occurred for out-of-phase ($\gamma = 0$: corresponds to $\beta = \pi$) and in-phase ($\gamma = \pi/2$: corresponds to $\beta = 0$) configurations, respectively. For both cases of Riley [10] and the present problem, the in-phase configuration yields minimum heat transfer. But the phase differences giving maximum heat transfer do not agree with each other. The dependency of heat transfer characteristics on the phase difference for the non-uniform wall temperatures is somewhat different from that for the imperfections of the wall shapes.

The dependency of heat transfer ($\overline{Nu}$) on the wave number k is shown in Fig. 4 with $Nu(k)$: $\overline{Nu} = Ra[Nu(k) \times \sin(\beta)]$. $Nu(k)$ has its maximum value at $k = k_m \approx 1.606$: as k increases, $Nu(k)$ is increased at $0 < k < k_m$, but is decreased at $k > k_m$ and approaches zero as $k \to \infty$.

Summarizing the mean Nusselt number dependencies on k and β, we can see that maximum heat transfer occurs at the wave number of $k \approx 1.606$ and phase difference of $\beta = \pi/2$, when the two walls have spatially periodic temperatures. Minimum heat transfer occurs

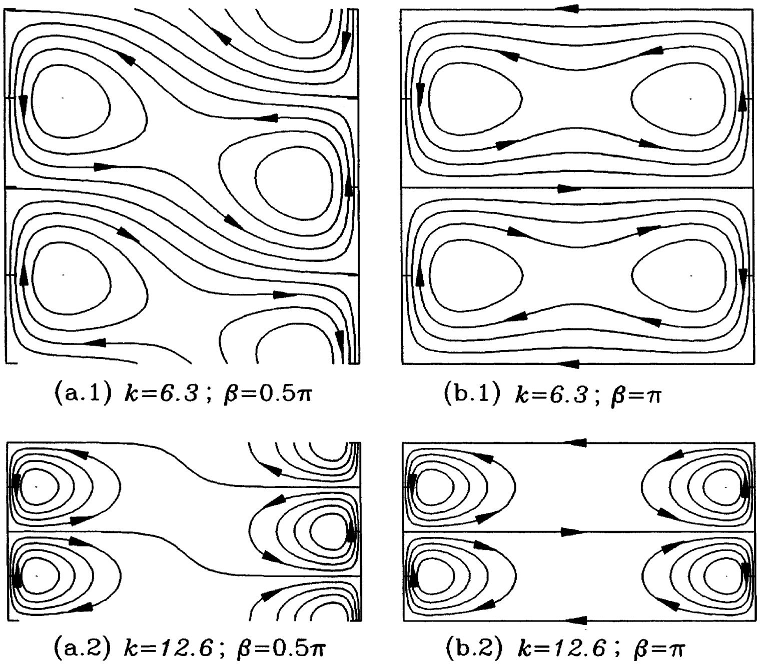

Fig. 3. Variation of flow fields with respect to k for $\beta = \pi/2$ and π: (a.1) $k = 6.3$ and $\beta = \pi/2$; (a.2) $k = 12.6$ and $\beta = \pi/2$; (b.1) $k = 6.3$ and $\beta = \pi$; (b.2) $k = 12.6$ and $\beta = \pi$. The range of x is $0 \leqslant x \leqslant 2\pi/k$.

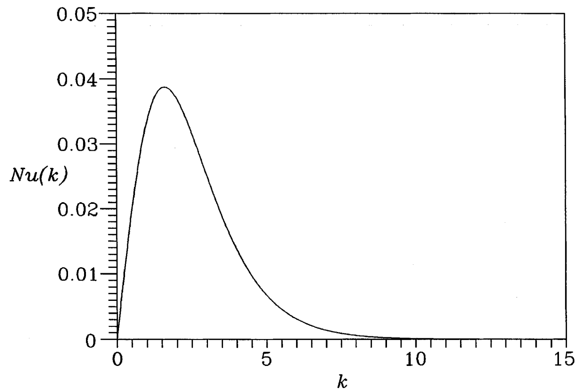

Fig. 4. Plot of $Nu(k)$ showing the dependency of mean Nusselt number ($\overline{Nu}$) on the wave number (k): $\overline{Nu} = Ra[Nu(k)\sin(\beta)]$.

for both in-phase ($\beta = 0$) and completely out-of-phase ($\beta = \pi$) configurations.

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